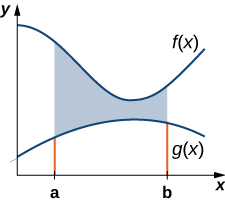
# Section 6.1: Areas Between Curves

We have seen how to calculate the area below a curve on a given interval. In this section, we expand that idea to calculate the area of more complex regions.

## Area of a Region between Two Curves

**Finding the Area between Two Curves**

Let and be continuous functions such that over an interval .



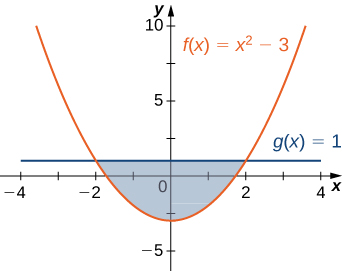
Let denote the region bounded above the graph of , below by the graph of , and on the left and right by the lines and , respectively. Then, the area of is given by

.

Media: Watch these [video1](https://youtu.be/q7De3oOp4Ug) and [video2](https://youtu.be/BH-V6Gx2n8E) examples on finding areas between curves.

Examples

1. Determine the area of the region between the two curves in the given figure by integrating over the -axis.



1. If is the region bounded above by the graph of the function and below by the graph of the function over the interval , find the area of region .
2. If is the region bounded above by the graph of the function and below by the graph of the function , find the area of region .

## Areas of Compound Regions

So far, we have required over the entire interval of interest, but often times the regions of interest are not simple.

**Finding the Area of a Region between Curves That Cross**

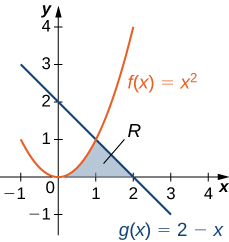
Let and be continuous functions over an interval . Let denote the region between the graphs of and , and be bounded on the left and right by the lines and , respectively. Then, the area of is given by

.

Media: Watch this [video](https://youtu.be/lskEXsJaJV0) example on finding areas with multiple regions.

Examples

1. If is the region between the graphs of the functions and over the interval , find the area of region .
2. Consider the region shown below. Find the area of .

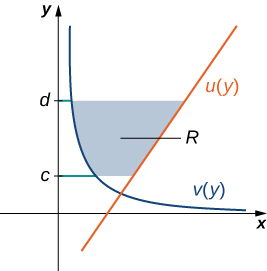


## Regions Defined with Respect to

We can also find the area between two graphs with respect to . Sometimes this method is easier to evaluate rather than evaluating multiple integrals to calculate the area of a region.

**Finding the Area between Two Curves, Integrating along the -axis**

Let and be continuous functions such that for all . Let denote the region bounded on the right by the graph of , on the left by the graph of , and above and below by the lines and , respectively.



Then, the area of is given by

.

Media: Watch this [video](https://youtu.be/70NQ3ISYihw) example on finding areas with respect to .

Example: Consider the region shown below. Integrate with respect to to find the area of .

